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Gordon D. Nigh

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**Prepared by**

Gordon D. Nigh
B.C.Ministry of Forests
Forest Science Program
712 Yates Street
Victoria, BC V8W 3E7

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## ABSTRACT

A suite of models consisting of height-age (site index), growth intercept, years-to-breast-height, juvenile height, and Site Index – Biogeoclimatic Ecosystem Classification (SIBEC) models are commonly used in British Columbia to estimate the height and site index of forest stands. Eighty plots of ponderosa pine stem analysis data were collected across the range of ponderosa pine in British Columbia. The years-to-breast-height, growth intercept, and juvenile height models were developed with these data. Height-age models were also developed, but are described elsewhere. The data were also used to further populate the SIBEC database. The growth intercept models were developed using standard techniques. The years-to-breast-height models used a slightly different functional form for the model. The juvenile height modelling technique was modified to more seamlessly splice the height curves into the height-age models.

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## INTRODUCTION

Recent research into methods of estimating site index and height growth has led to the development of a suite of models: height-age (site index), growth intercept, years-to-breast-height, Site Index – Biogeoclimatic Ecosystem Classification (SIBEC), and most recently, juvenile height models.

- The height-age models are the core of the suite of models. They estimate height from site index and breast height age, and are also used to estimate site index from height and breast height age using an iterative numerical technique.
- The growth intercept models estimate site index from height and breast height age for site trees that are between breast height age 1 and 50. Their accuracy declines for younger site trees, so it is recommended that the site trees be at least 5 years breast height age. Generally, the growth intercept models provide the most accurate estimates of site index for trees between 5 and 50 years breast height age.
- The years-to-breast-height models estimate the number of years it takes a tree to grow from germination to breast height (1.3 m) as a function of site index. They are used to convert total age into breast height age and vice versa.
- The SIBEC model provides an estimate of the average site index for a given species growing on a specific ecosystem (site series).
- Juvenile height models estimate tree heights from germination up to 1.3 m or more. The original intent was to get good height estimates for regenerating stands. However, the models were extended to a height of 3 m to estimate green-up age. The cut-off age for the juvenile height models is arbitrary, and depends to some extent on the range of the data.

This technical report describes the development of growth intercept, years-to-breast-height, and juvenile height models for ponderosa pine (*Pinus ponderosa* Dougl. ex Laws). Height models for this species are described in Nigh (2003). Although a SIBEC model is not developed with data from this project, these data are used to further populate the SIBEC database.

## DATA

The data for these models are more fully described in Nigh (2003). Eighty ponderosa pine 0.01 ha stem analysis plots were successfully established throughout the range of ponderosa pine in British Columbia in mature (> 50 years breast height age) stands. One ponderosa pine site tree was identified and sampled in each plot. The tree was felled and its stem was split open to reveal the pith from the point of germination up to where the stem diameter is about 3–4 cm. The annual height growth of the tree was measured from the exposed pith nodes, which identify the termination of height growth each year. In the upper portion of the stem where splitting was not possible, height growth was identified from the annual branch whorls. This resulted in site tree heights from total age 0 up to the total age of the tree for 80 plots.

The total ages for the trees were converted into breast height ages by subtracting the age of the pith node immediately below breast height from the age of the pith nodes above breast height. The breast height age for the pith node immediately below 1.3 m is 0, and is undefined for lower pith nodes. The site index for the plot is, by definition, the height of the tree at breast height age 50.

The following describes the analysis of the three models. In each case, the standard regression assumptions were tested (Ratkowsky 1983; Sen and Srivastava 1990).

**Years-to-Breast-Height Model**

The number of years that it takes a tree to reach breast height is calculated using equation (1),

$$YTBH = A_0 + \frac{1.3 - H_0}{H_1 - H_0}, \quad (1)$$

where: $YTBH$ is years-to-breast-height, $A_0$ is the total age (years) of the pith node immediately below breast height, $H_0$ is the height (m) of the node immediately below breast height, and $H_1$ is the height (m) of the tree at breast height age 1. The second term in the right-hand side of (1) adjusts for a partial year's growth to reach breast height. The years-to-breast-height model is shown in equation (2),

$$YTBH = a_0 \times a_1^{SI} + \varepsilon, \quad (2)$$

where: $a_0$ and $a_1$ are model parameters, $SI$ is site index (m @ breast height age 50), and $\varepsilon$ is a random error term. This model was fit as a weighted (to induce a constant variance) nonlinear mixed effects model with site index as a random effect. Because weighting makes the root mean squared error difficult to interpret, the unweighted errors (actual – predicted years-to-breast-height) were calculated and then the mean error and its standard deviation were calculated.

**Growth Intercept Model**

The growth intercept was determined for ages 1 – 50 for each plot using equation (3),

$$GI = \frac{H - 1.3}{A - \frac{1.3 - H}{H_1 - H_0}} \times 100, \quad (3)$$

where: $GI$ is the growth intercept (cm/yr) and $H$ is tree height (m) at breast height age $A$ (yr). The growth intercept is the average annual height growth (cm/yr) for the site tree. An adjustment is made to the age in the denominator because the first year's growth above breast height is only a partial year's growth and hence is pro-rated. The growth intercept model is given in equation (4),

$$SI = 1.3 + a_0 \times GI^{a_1} + \varepsilon. \quad (4)$$

A separate model is fit to the data for ages 1 – 50 inclusive using nonlinear least squares regression. This resulted in 50 sets of parameters for the growth intercept model, one set for each age between 1 and 50, which is the age range

for the growth intercept model. The parameter set corresponding to the breast height age of the sample tree is used in the model for estimating site index.

**Juvenile Height Model**

The juvenile height model is given by equation (5),

$$H = a_0 \times A^{a_1} \times a_2^{A} + \varepsilon \,. \tag{5}$$

This model was conditioned to return a height estimate of 1.3 m at the years-to-breast-height. This conditioning provides a smooth transition to the height-age model (Nigh 2003) at a height of 1.3 m. The conditioning was done by expressing parameter $a_0$ as a function of age and years-to-breast-height, as shown in equation (6),

$$a_0 = \frac{1.3}{YTBH^{a_1} \times a_2^{YTBH}}. \tag{6}$$

Predictions from equation (2) were used in place of actual years-to-breast-height measurements and site index was treated as a random effect. Parameters $a_1$ and $a_2$ were estimated using nonlinear mixed effects modelling software.

## RESULTS

**Years-to-Breast-Height Model**

The estimated values for parameters $a_0$ and $a_1$ and their standard errors (in parentheses) are 36.35 (4.20) and 0.9318 (0.00747), respectively. The mean error is 0.26 years, which was not significantly different from 0, and the standard deviation of the errors is 4.8 years.

**Growth Intercept Model**

The 50 sets of parameter estimates and the root mean squared errors for the growth intercept models are shown in Table 1.

**Juvenile Height Model**

The estimates for parameters $a_1$ and $a_2$, and their standard errors (in parentheses), are 1.137 (0.0278) and 1.016 (0.00245), respectively. The root mean squared error for this model is 0.055 m, which is quite small.

## DISCUSSION

The fitted years-to-breast-height, growth intercept, and juvenile height models are given in equations (7), (8), and (9), respectively.

$$YTBH = 36.35 \times 0.9318^{SI}; \tag{7}$$

$$SI = 1.3 + a_0 \times GI^{a_1}; \tag{8}$$

where parameters $a_0$ and $a_1$ are given in Table 1 for breast height ages 1 – 50, inclusive; and

$$H = \frac{1.3 \times A^{1.137} \times 1.016^{A}}{YTBH^{1.137} \times 1.016^{YTBH}}. \tag{9}$$

TABLE 1 *Parameter estimates, their standard errors, and the root mean squared error for the 50 growth intercept models*

| BHA (yr) | Parameter $a_0$ | $a_1$ | RMSE | BHA (yr) | Parameter $a_0$ | $a_1$ | RMSE |
|---|---|---|---|---|---|---|---|
| 1 | 5.631 | 0.2745 | 3.546 | 26 | 1.520 | 0.6771 | 1.641 |
| 2 | 4.381 | 0.3633 | 3.202 | 27 | 1.446 | 0.6914 | 1.602 |
| 3 | 3.791 | 0.4127 | 3.086 | 28 | 1.397 | 0.7012 | 1.548 |
| 4 | 3.350 | 0.4545 | 2.995 | 29 | 1.340 | 0.7130 | 1.481 |
| 5 | 3.076 | 0.4820 | 2.878 | 30 | 1.275 | 0.7271 | 1.407 |
| 6 | 2.979 | 0.4928 | 2.802 | 31 | 1.195 | 0.7457 | 1.333 |
| 7 | 2.828 | 0.5095 | 2.737 | 32 | 1.132 | 0.7618 | 1.283 |
| 8 | 2.699 | 0.5233 | 2.697 | 33 | 1.066 | 0.7793 | 1.242 |
| 9 | 2.647 | 0.5277 | 2.649 | 34 | 1.005 | 0.7965 | 1.152 |
| 10 | 2.613 | 0.5301 | 2.591 | 35 | 0.9542 | 0.8114 | 1.081 |
| 11 | 2.539 | 0.5367 | 2.492 | 36 | 0.9156 | 0.8231 | 1.013 |
| 12 | 2.490 | 0.5409 | 2.420 | 37 | 0.8797 | 0.8343 | 0.9601 |
| 13 | 2.410 | 0.5499 | 2.353 | 38 | 0.8241 | 0.8528 | 0.8956 |
| 14 | 2.309 | 0.5624 | 2.253 | 49 | 0.7806 | 0.8682 | 0.8216 |
| 15 | 2.247 | 0.5685 | 2.184 | 40 | 0.7372 | 0.8844 | 0.7571 |
| 16 | 2.193 | 0.5745 | 2.165 | 41 | 0.6979 | 0.9000 | 0.6819 |
| 17 | 2.122 | 0.5842 | 2.118 | 42 | 0.6669 | 0.9130 | 0.6127 |
| 18 | 2.013 | 0.5998 | 2.068 | 43 | 0.6360 | 0.9266 | 0.5578 |
| 19 | 1.950 | 0.6088 | 2.037 | 44 | 0.6046 | 0.9414 | 0.4892 |
| 20 | 1.896 | 0.6159 | 1.989 | 45 | 0.5801 | 0.9535 | 0.4161 |
| 21 | 1.827 | 0.6263 | 1.937 | 46 | 0.5585 | 0.9643 | 0.3423 |
| 22 | 1.746 | 0.6391 | 1.855 | 47 | 0.5467 | 0.9707 | 0.2744 |
| 23 | 1.680 | 0.6496 | 1.799 | 48 | 0.5338 | 0.9778 | 0.2099 |
| 24 | 1.629 | 0.6582 | 1.755 | 49 | 0.5139 | 0.9891 | 0.1411 |
| 25 | 1.582 | 0.6660 | 1.699 | 50 | 0.4978 | 0.9984 | 0.06919 |

BHA = breast height age (yr); $a_0$, $a_1$ = model parameters;
RMSE = root mean squared error.

The years-to-breast-height model has a different functional form from many of the years-to-breast-height models in British Columbia (Thrower et al. 1994; Nussbaum 1996). However, the root mean squared error for these models is comparable to the standard deviation for this model (4.8 years). The height growth of trees varies considerably below breast height, which is reflected in the root mean squared error (or the standard deviation, in this situation) associated with years-to-breast-height models in general. This result supports the use of breast height age site index models instead of total age models because the breast height age models avoid the early erratic growth below breast height.

The growth intercept models are standard for the type used in British Columbia. The last change to the modelling technique and functional form of the model is documented in Nigh (1997a). These models work well and are widely accepted. The accuracy of the ponderosa pine models are comparable to other interior and coastal species (Nigh 1997b, 1999), but are not as good as those for lodgepole pine (*Pinus contorta* var. *latifolia*) (Nigh 1997c).

The juvenile height growth modelling technique differs slightly from past efforts (see, for example, Nigh and Love 2000), which used age and site index to predict juvenile height growth up to some age that depends on the age range of the model calibration data. These curves were eventu-

ally spliced into the height-age models, which caused problems with smoothness (Nigh and Polsson 2002). Because height-age and juvenile height models were developed with the same set of data, the curves could be spliced together more seamlessly by conditioning the juvenile height model, as previously described. Consequently, juvenile height growth is now estimated from total age and years-to-breast-height. Years-to-breast-height will either be known or, more commonly, estimated from site index with equation (7). Figure 1 shows the juvenile height model (below a height of 1.3 m) spliced to the height-age model above 1 m reported by Nigh (2003). The transition between the two models is relatively seamless, and avoids the lengthy calculations and analyses of the past.

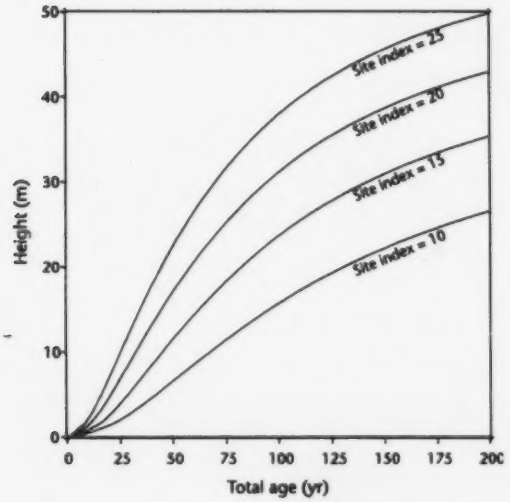


FIGURE 1 *Juvenile height model (below 1.3 m) and the height-age model (above 1.3 m) for site indices 10, 15, 20, and 25 m.*

## CONCLUSIONS

The years-to-breast-height, growth intercept, and juvenile height models presented here are part of a suite of models that estimate the height and site index of ponderosa pine. These models are most suitable for stands up to 1.3 m tall, or up to breast height age 50 in the case of the growth intercept models. They should be used in conjunction with the height-age model for ponderosa pine for older or taller stands. These models will be useful for estimating site index in juvenile stands, early height growth patterns, and the number of years it takes a stand to reach breast height or green-up. This project also provides site index data for the SIBEC database.

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